What if the gravitational constant *G* is not a true constant?

Lakshmi Kantha[a)]
ECAE 117, Engineering Building, Colorado Ave., University of Colorado, Boulder, Colorado 80309-0431, USA and Institute of Marine Sciences, CNR-ISMAR, Arsenale-Tesa 104, Castello 2737/F, I-30122, Venice, Italy

(Received 4 July 2011; accepted 12 April 2012; published online 3 July 2012)

Abstract: It is universally accepted that there are three fundamental physical constants in the universe, Newton's universal gravitational constant G, Planck's constant $\hbar$, and the speed of light c. The constancy of G is built into the existing models of cosmic evolution, including the Big Bang model of the universe. Although there is little doubt that $\hbar$ and c are universal constants, the same degree of certainty cannot be attributed to G. So an interesting question is: What if G is not a true constant but is a function of cosmic time? In this short note, we explore the consequences of a nonconstant G on the cosmos. The proposed nonconstant G model of the cosmos is consistent with modern observations of the cosmic evolution, which strongly support a still-accelerating universe dominated by dark energy, requiring a nonzero cosmological constant Λ in Einstein's Field Equation. Although there is incontrovertible evidence from the Wilkinson microwave anisotropy probe (WMAP) that the universe is very nearly flat, current Big Bang models require the cosmological constant to remain unchanged as the universe expands. The model proposed here allows for a changing G and requires the cosmological constant to decrease with cosmic time as the universe expands. In all other aspects, it is consistent with modern observations and existing Big Bang models. © *2012 Physics Essays Publication.* [DOI: 10.4006/0836-1398-25.2.282]

Résumé: Il est universellement reconnu qu'il ya trois constantes physiques fondamentales dans l'univers, la constante gravitationnelle universelle G de Newton, la constante de Planck $\hbar$, et la vitesse de la lumière c. La constance de G est intégrée dans les modèles existants de l'évolution cosmique, y compris le modèle du Big Bang de l'univers. Bien qu'il y ait peu de doute que $\hbar$ et c soient des constantes universelles, le même degré de certitude ne peut être attribué à G. Il y a donc une question intéressante: Que faire si G n'est pas une vraie constante, mais est une fonction du temps cosmique? Dans cette courte note, nous explorons les conséquences d'un G non constant sur le cosmos. Le modèle G proposé non constant du cosmos est en accord avec les observations modernes de l'évolution cosmique, qui soutiennent fortement l'idée d'un univers encore dominé par l'accélération de l'énergie sombre, nécessitant une constante cosmologique Λ non-nulle dans l'équation du champ d'Einstein. Bien qu'il y ait des preuves irréfutables de WMAP que l'univers soit presque complètement plat, les modèles actuels du Big Bang requièrent que la constante cosmologique reste inchangée tant que l'univers se dilate. Le modèle proposé ici permet de changer G et exige que la constante cosmologique diminue avec le temps cosmique tant que l'univers se dilate. Dans tous les autres aspects, il est cohérent avec les observations modernes et des modèles existants du Big Bang.

Key words: Big Bang Model; Cosmological Constant; Einstein's Field Equation; Newton's Gravitational Constant; Cosmic Evolution; Fundamental Physical Constants; Models of the Universe; Inflation Theory; Nucleosynthesis; Cosmic Microwave Background.

I. INTRODUCTION

It is universally accepted that there are three fundamental physical constants in the universe: the speed of light c ($\sim 2.998\times 10^{8}$ m s^{-1}), Planck's constant $\hbar$ ($\sim 1.055\times 10^{-34}$ m^{2}s^{-1} kg) and Newton's gravitational constant G ($\sim 6.674\times 10^{-11}$ m^{3}s^{-2} kg^{-1}). The constancy of G is built into existing models of cosmic evolution, including the Big Bang model. Although there is little doubt that $\hbar$ and c are universal constants, the same degree of certainty cannot be attributed to G. There have been some suggestions in literature, starting with Dirac,[1,2] that G may not be a true constant. So an interesting question is: What if G is not a true constant, but a function of cosmic time? What are the implications to the cosmic evolution?

As Max Planck pointed out, it is possible to define fundamental units of length, mass, and time based on these constants: $\ell_P=(\hbar G/c^3)^{1/2}$, $m_P=(\hbar c/G)^{1/2}$, and $t_P=(\hbar G/c^5)^{1/2}$, called Planck length, mass, and time, respectively. However, note that it is impossible to form a

[a)]kantha@colorado.edu

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dimensionless number out of these three constants: Another fundamental constant, the mass of a nucleon, m_n, must be brought in. $\mathbb{N}=Gm_n^2/\hbar c$ is dimensionless and therefore a variable G implies variable $\mathbb{N}$. The current value of $\mathbb{N}$ is 5.89×10^{-39}. It also means that the Planck–Boltzmann temperature defined by Kirwan,[3] $\Im=(\hbar c^5/Gk_B^2)1/2$, where $k_B=1.3806503\times10^{-23}$ m^2 kg s^{-2} K^{-1}, would also be a function of cosmic time. Current value of $\Im$ is 1.42×10^{32}.

Since Dirac's original work, there has been sporadic interest in variable G models of the Cosmos[4–8] (see also the references in Singh and Singh[4] for a list of earlier work on the topic). Momeni's work[5] on Brownian motion suggests that G decreases with time. He also summarizes the plausible values of $\dot{G}/G$ from recent observations. Singh[6] considered time-dependent G and Λ in the early inflationary and radiation-dominated universe and suggested a model in which $\Lambda\sim t^{-2}$ but G could be either proportional or inversely proportional to t. Many other investigators have considered time-dependent Λ and hence time-dependent G, including Jamil and Debnath,[7] who present a model in which $\Lambda \sim H^2$, where H is the Hubble parameter. Singh and Singh[4] explore a nonconstant G cosmological model with $\Lambda \sim H^2$ also, but with matter in the form of a viscous fluid. However, to our knowledge, no one has explored the possibility of matching the radiation-dominated epoch to the matter-dominated epoch in the context of a variable G.

In the interest of readers not familiar with the subject and to put the new hypothesis in a proper context, we provide here a brief summary of existing models of the universe. Einstein's Field Equation (EFE), which is the basis of all models of cosmic evolution, can be written as (including the cosmological constant Λ Einstein introduced in an ad hoc manner but withdrew later):

$$R_{ij}-(1/2)Rg_{ij}-\Lambda g_{ij}=G_{ij}-\Lambda g_{ij}=(8\pi G/c^4)S_{ij}, \qquad (1)$$

where R_{ij} is the Ricci curvature tensor (units of m^{-2}), g_{ij} is the metric tensor, R is the scalar curvature (units of m^{-2}), and S_{ij} is the stress-energy tensor (units of Pa or Nm^{-2}); Λ has units of m^{-2} and $8\pi G/c^4$ has units of N^{-1}. It is important to note that the derivation of Eq. (1) does not require the assumption that G is a constant.

It is well known that Einstein added the cosmological constant term in Eq. (1) to obtain a steady-state universe, but once Hubble's observations proved that the universe is expanding, he deleted it, asserting that it was the greatest blunder he ever made. Incidentally, we aerodynamicists know that Einstein's airfoil, derived with no regard to the behavior of the boundary layer in an adverse pressure gradient, was a real mistake; the cosmological constant was not. However, recent observations suggest that the universe is not only expanding, but the expansion rate is increasing. An accelerating universe requires the reintroduction of the cosmological constant term in EFE.

II. COSMOLOGICAL EVOLUTION

The two basic equations governing the evolution of the cosmos are[9]

$$\left(\frac{\dot{R}}{R}\right)^2+\frac{kc^2}{R^2}-\frac{\Lambda c^2}{3}=\frac{8\pi G}{3}\rho, \qquad (2)$$

$$\frac{2\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2+\frac{kc^2}{R^2}-\Lambda c^2=-\frac{8\pi G}{c^2}p, \qquad (3)$$

where k is the curvature of space, ρ is the density, and p is the pressure. Dots denote derivative with respect to cosmological time t. Equation (2) is the Friedmann equation. Note that neither Eq. (2) nor Eq. (3) involves the assumption G = constant, and so both equations are valid for a time-dependent G also. Substituting from Eq. (2), Eq. (3) can also be written as

$$\frac{2\ddot{R}}{R}=\frac{2}{3}\Lambda c^2-\frac{8\pi G}{3}\left(\rho+3\frac{p}{c^2}\right). \qquad (4)$$

Because the term on the right-hand side of Eq. (4) involving G is negative definite (at least for the post-inflation period), without the cosmological term, $\ddot{R}<0$, and the universe would decelerate. In other words, gravitation leads to a decelerating universe, but modern observations indicate a still-accelerating universe. This of course means that the cosmological constant must be positive and big enough to overcome the effect of gravitational attraction.

The mass conservation equation is[9]

$$\dot{\rho}+3(\rho+p/c^2)(\dot{R}/R)=0. \qquad (5)$$

Equation (5) can be derived from Eqs. (2) and (3) by differentiating Eq. (2) to obtain $\ddot{R}$ and using it to eliminate $\ddot{R}$ from Eq. (3). The derivation, however, requires that the cosmological parameters Λ and k and the Newtonian gravitational parameter G not be functions of time. If they are, Eq. (5) must involve the time rate of change of these quantities. The equation of state is assumed to be of the form

$$p=w\rho c^2, \qquad (6)$$

where $w=1/3$ for relativistic particles (radiation) and hence radiation-dominated early universe, and $w=0$ for nonrelativistic particles (dust) and hence matter-dominated universe. Also, $w=1$ for a stiff fluid and -1 for a vacuum fluid (during inflation). With the use of Eq. (6), Eq. (5) becomes

$$\dot{\rho}/\rho=-3(1+w)(\dot{R}/R). \qquad (7)$$

Integration of Eq. (7) yields

$$\rho\sim R^{-3(1+w)}\sim R^{-n}, \qquad (8)$$

where

$$n=3(1+w). \qquad (9)$$

Note that $n=0$ during inflation.

For the matter-dominated universe ($n = 3$),

$$\rho \sim R^{-3}.$$

For the radiation-dominated universe ($n = 4$),

$$\rho \sim R^{-4}. \tag{11}$$

Because

$$\sigma T^4 \sim \rho c^3, \tag{12}$$

where T is the temperature of the universe and σ is Stefan–Boltzmann constant ($\sim 5.6704 \times 10^{-8}\,\mathrm{W\,m^{-2}}\,K^{-4}$). For radiation-dominated early universe, $T \sim R^{-1}$. This equation is the basis[4] for the Big Bang nucleosynthesis (BBN) and other aspects of the early evolution of the universe during the Big Bang.

Interestingly, the equation of state during the "inflation" phase of the Big Bang, during which the universe expanded exponentially by a factor of 10^{26} in an infinitesimal time ($\sim 10^{-35}$ s) from a size smaller than an atomic nucleus to a macroscopic size of the order of a meter, must be such as to make the gravitational term on the right-hand side of Eq. (4) positive.[10] The proposed equation of state during inflation is

$$p = V(\phi) = -\rho c^2, \tag{13}$$

so that $w=-1$ and therefore $n=0$ and ρ=constant during the inflationary phase.

III. MODELS OF THE UNIVERSE

Hubble's law states that the velocity of stars and galaxies in the universe is given by

$$v = H(t) d, \tag{14}$$

where $H = \dot{R}/R$. Substituting into Eq. (2), we get

$$H^2 + \frac{kc^2}{R^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho. \tag{15}$$

The value of H at the present epoch, H_0 is known as the Hubble constant and immense efforts have gone into measuring it accurately. The most accurate measurements are from the Wilkinson microwave anisotropy probe (WMAP), which gives a value of 73.5 ± 3.2 km s^{-1} Mpc^{-1}. Combined with other recent and advanced astronomical measurements,[11] we get

$$H_0 \sim 70.8 \pm 1.6\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \sim 2.2945 \times 10^{-18}\,\mathrm{s^{-1}}, \tag{16}$$

where $1\,\mathrm{Mpc} = 3.08568 \times 10^{22}$ m. Equation (15) can be written as

$$H^2 = H_0^2 \left[\left(\frac{G}{G_0}\right)\left(\frac{\rho_0}{\rho_{cr}}\right)\left(\frac{\rho}{\rho_0}\right) + \frac{\Lambda_0 c^2}{3H_0^2}\left(\frac{\Lambda}{\Lambda_0}\right) - \frac{kc^2}{H_0^2 R_0^2}\left(\frac{R_0}{R}\right)^2 \right], \tag{17}$$

where subscript 0 denotes the current value (note that we have introduced the subscript 0 for G and Λ, to allow for the possibility that they may be functions of cosmological time t). The critical density of the universe has been defined as

$$\rho_{\mathrm{cr}} = \frac{3H_0^2}{8\pi G_0}. \tag{18}$$

$\rho_{\mathrm{cr}} \sim 9.416 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$, based on the most recent values for G_0 and H_0. The Hubble constant H_0 and the critical density ρ_{cr} are the two most important parameters in cosmic evolution. If we put

$$\Omega_m = \frac{\rho_0}{\rho_{\mathrm{cr}}}, \qquad \Omega_k = \frac{kc^2}{H_0^2 R_0^2}, \qquad \Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}, \tag{19}$$

we get

$$H = \frac{\dot{R}}{R} = H_0 \left[\left(\frac{G}{G_0}\right)\Omega_m \left(\frac{\rho}{\rho_0}\right) - \Omega_k \left(\frac{R_0}{R}\right)^2 + \Omega_\Lambda \right]^{1/2}. \tag{20}$$

Normalizing Eq. (20),

$$\Re = R/R_0, \qquad \tau = t/t_o, \tag{21}$$

and choosing the normalizing time t_o so that

$$t_o = \frac{2}{3H_0} \tag{22}$$

gives

$$\frac{d\Re}{d\tau} = \frac{2}{3}\sqrt{\Omega_m}\,\Re \left[\left(\frac{G}{G_0}\right)\left(\frac{\rho}{\rho_0}\right) - \frac{\Omega_k}{\Omega_m}\Re^{-2} + \frac{\Omega_\Lambda}{\Omega_m} \right]^{1/2}. \tag{23}$$

With the best known value for H_0,

$$t_o \sim 9.2071\,\mathrm{Gyr}\,(2.906 \times 10^{17}\,\mathrm{s}) \tag{24}$$

Because

$$\rho/\rho_0 = (R_0/R)^n = \Re^{-n}, \tag{25}$$

Eq. (23) becomes

$$\frac{d\Re}{d\tau} = \frac{2}{3}\sqrt{\Omega_m}\,\Re \left[\left(\frac{G}{G_0}\right)\Re^{-n} - \frac{\Omega_k}{\Omega_m}\Re^{-2} + \frac{\Omega_\Lambda}{\Omega_m} \right]^{1/2}, \tag{26}$$

and Eq. (20) holds for both radiation- and matter-dominated universes. Note also that

$$R_0/R = \Re^{-1} = 1 + z, \tag{27}$$

where z is the redshift. Equation (27) can be used to express Eqs. (20) and (26) in terms of the redshift, which can be measured accurately.

A. Matter-dominated universe

For the matter-dominated universe $n = 3$, and Eqs. (20) and (25) give

$$H = \frac{\dot{R}}{R} = H_0 \left[\left(\frac{G}{G_0}\right)\Omega_m \left(\frac{R_0}{R}\right)^3 - \Omega_k \left(\frac{R_0}{R}\right)^2 + \Omega_\Lambda \right]^{1/2}. \tag{28}$$

Applying Eq. (28) to the present epoch ($H = H_0, G = G_0$,

$R = R_0$) gives

$$\Omega_m - \Omega_k + \Omega_\Lambda = 1, \tag{29}$$

where superscript 0 that would indicate the current epoch has been omitted for simplicity. Equation (26) becomes

$$\frac{d\Re}{d\tau} = \frac{2}{3}\sqrt{\Omega_m}\Re\left[\left(\frac{G}{G_0}\right)\Re^{-3} - \frac{\Omega_k}{\Omega_m}\Re^{-2} + \frac{\Omega_\Lambda}{\Omega_m}\right]^{1/2}. \tag{30}$$

Note that for a flat universe ($\Omega_k = 0$) preferred by most cosmologists and supported by modern observations, such as the angular fluctuation spectrum from WMAP,

$$\frac{d\Re}{d\tau} = \frac{2}{3}\sqrt{\Omega_m}\Re\left[\left(\frac{G}{G_0}\right)\Re^{-3} + \frac{\Omega_\Lambda}{\Omega_m}\right]^{1/2}, \tag{31}$$

where

$$\Omega_m + \Omega_\Lambda = 1, \tag{32}$$

with the additional condition $\Re = 1$ for

$$\tau = \tau_0 = t_U / t_o. \tag{33}$$

If we assume $G/G_0 = 1$, then most recent observations of the cosmos such as WMAP appear to be consistent with

$$\Omega_k = 0, \qquad \Omega_m = 0.27, \qquad \Omega_\Lambda = 0.73. \tag{34}$$

1. Einstein–de Sitter universe model

The Einstein–de Sitter (EdS) universe requires

$$\Omega_m = 1, \qquad \Omega_\Lambda = 0, \qquad \Omega_k = 0, \tag{35}$$

and therefore for the EdS universe, putting $G/G_0 = 1$, Eq. (30) becomes

$$d\Re/d\tau = (2/3)\Re^{-1/2}, \tag{36}$$

so that $\Re^{3/2} = \tau$ or, equivalently,

$$\Re = \tau^{2/3}. \tag{37}$$

The current age of the EdS universe $t_U = t_o \sim 9.207$ Gyr (2.906×10^{17} s). This does not fit well with observations. There appear to be galactic clusters that are older than this age, which suggests that the age of the universe is much higher and EdS model is incorrect.

Incidentally, Einstein used Eq. (3) and invoked a static universe by putting $\dot{R} = \ddot{R} = 0$ and $\Lambda = k/R^2$ or equivalently, $\Omega_k = 3\Omega_\Lambda$, to obtain the radius of a static universe $R_{SU} = (k/\Lambda)^{1/2}$. Substituting in Eq. (2) gives $\Lambda c^2 = 4\pi G_0 \rho_0$ or $\Omega_\Lambda = 0.5\Omega_m$. Thus Einstein's static universe requires

$$\Omega_m = 1, \qquad \Omega_\Lambda = 0.5, \qquad \Omega_k = 1.5. \tag{38}$$

2. Current models of the universe

The solution of Eq. (31) for a flat universe ($\Omega_k = 0$), assuming $G/G_0 = 1$, is

$$\Re = (\Omega_m/\Omega_\Lambda)^{1/3}\left[\sinh(\sqrt{\Omega_\Lambda}\cdot\tau)\right]^{2/3} \tag{39}$$

or

$$\tau = 1/\sqrt{\Omega_\Lambda}\,\sinh^{-1}(\sqrt{\Omega_\Lambda/\Omega_m}\Re^{3/2}), \tag{40}$$

so that the current age of the universe obtained by putting $\Re = 1$ at $\tau = \tau_0$ in Eq. (40) is

$$t_U = \tau_0 t_o, \tag{41}$$

where

$$\tau_0 = 1/\sqrt{\Omega_\Lambda}\,\sinh^{-1}(\sqrt{\Omega_\Lambda/\Omega_m}).$$

With the use of WMAP observations, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$ (Eq. 34), and $\tau_0 = 1.489$. Therefore,

$$t_U \sim 13.71\text{Gyr}(4.327 \times 10^{17}\,\text{s}) \tag{43}$$

is the current age of the universe. This larger value is consistent with astronomical observations of the age of galaxy clusters.

B. Radiation-dominated universe

For radiation-dominated universe $n = 4$, and Eqs. (20) and (25) give

$$H = \frac{\dot{R}}{R} = H_0\left[\left(\frac{G}{G_0}\right)\Omega_m\left(\frac{R_0}{R}\right)^4 - \Omega_k\left(\frac{R_0}{R}\right)^2 + \Omega_\Lambda\right]^{1/2}. \tag{44}$$

Equation (26) becomes

$$\frac{d\Re}{d\tau} = \frac{2}{3}\sqrt{\Omega_m}\Re\left[\left(\frac{G}{G_0}\right)\Re^{-4} - \frac{\Omega_k}{\Omega_m}\Re^{-2} + \frac{\Omega_\Lambda}{\Omega_m}\right]^{1/2}. \tag{45}$$

For a flat universe ($\Omega_\Lambda = 0$), and $\Omega_\Lambda = 0$,

$$\frac{d}{d\tau}(\Re^2) = \frac{4\sqrt{\Omega_m}}{3}\sqrt{\frac{G}{G_0}}.$$

If G is also constant, $G/G_0 = 1$ and

$$\Re = (16\Omega_m/9)^{1/4}\tau^{1/2}. \tag{46}$$

Because $\rho = (4\sigma/c^3)T^4$ and $\rho = \rho_0\Re^{-4}$

$$T = \left(\frac{\rho_0 c^3}{4\sigma}\right)^{1/4},$$

$$\Re^{-1} = \left(\frac{3H_0^2 c^3 \Omega_m}{32\pi G_0 \sigma}\right)^{1/4},$$

$$\Re^{-1} = \left(\frac{27H_0^2 c^3}{512\pi G_0 \sigma}\right)^{1/4}\tau^{-1/2}. \tag{47}$$

Therefore

$$\Re \sim \tau^{1/2}, \qquad T \sim \Re^{-1} \sim \tau^{-1/2}. \tag{48}$$

This implies that $T \to \infty$ as $\tau \to 0$, and this fact alone was the death knell for the steady-state universe model. From a very high value immediately after inflation, the universe is thought to have cooled down to temperatures of the order of 10^9–10^{10} K soon after (2–15 min) the Big Bang, enabling BBN to occur quite early in the radiation-dominated universe. However, the universe was then filled with free electrons, which provided a scattering medium for radiation, and therefore the universe was opaque. As the universe cooled, electrons began to bind themselves to protons electrostatically, and at around $T \sim 2967$ K, the number of free electrons dropped precipitously. Without scattering by electrons, radiation could now escape freely and the universe became transparent.[9] This recombination is thought to have occurred 0.38 Myr ($\tau \sim 0.000041$) after the Big Bang. Stars started forming around 0.3 Gyr ($\tau \sim 0.0326$) and galaxies started forming around 1 Gyr ($\tau \sim 0.109$). Note that $T_0 \sim 2.728$Kor the cosmic microwave background (CMB) blackbody temperature, as deduced from the COBE and the more recent highly accurate WMAP spectrum measurements.

C. Inflation

During inflation, $n=0$, and for a flat universe ($\Omega_k=0$), Eqs. (23) and (25) give

$$\left(\frac{\Re'}{\Re}\right)^2=\frac{4}{9}\left[\left(\frac{G}{G_0}\right)\Omega_m+\Omega_\Lambda\right]. \tag{49}$$

Equation (4) becomes

$$\frac{\Re''}{\Re}=\frac{4}{9}\left[\left(\frac{G}{G_0}\right)\Omega_m+\Omega_\Lambda\right]. \tag{50}$$

Therefore

$$\Re''/\Re=(\Re'/\Re)^2, \tag{51}$$

whose solution is

$$R=R_i\exp\left(\int_{\tau_i}^{\tau} H\,d\tau\right). \tag{52}$$

This exponential evolution[5] enabled the Universe to expand from a small homogeneous patch no bigger than 10^{-26} m to about a meter in about 10^{-35} s. Note that the solution [Eq. (52)] holds for nonconstant G and Ω_Λ also.

IV. WHAT IF?

Note that the current models of the universe assume G and Ω_Λ are constant. However, there is no reason why G should remain unchanged as the universe expands. Paul Dirac[1] was one of the very first to suggest that G could be a function of cosmological time. So let us look for alternative solutions involving a time-varying G.

Time-varying G also requires time-varying cosmological parameters, and using Eqs. (2) and (3), it is easy to show that for vanishing of the Einstein tensor (mass conservation)

$$\dot\rho+3\left(\rho+\frac{p}{c^2}\right)\left(\frac{\dot R}{R}\right)+\frac{1}{G}\left[\dot G\rho+\frac{\dot\Lambda c^2}{8\pi}-\frac{3\dot k c^2}{8\pi R^2}\right]=0 \tag{53}$$

instead of Eq. (5). Note that k is zero for a flat universe and the $\dot k$ term in Eq. (53) must be zero. If the terms in square brackets in Eq. (53) sum to zero, Eq. (5) remains the equation of state and the energy conservation is satisfied. Thus to zeroth order, we will put these terms to zero in the proposed nonconstant G model. This means

$$\Omega'_\Lambda=-\Omega_m^0(\rho/\rho_0)f', \tag{54}$$

or, equivalently,

$$\boxed{\Omega'_\Lambda=-\Omega_m^0\Re^{-n}f'}, \tag{55}$$

where

$$\frac{G}{G_0}=f(\tau), \tag{56}$$

and prime denotes the derivative with respect to τ. Equation (4) can be rewritten as

$$\frac{\Re''}{\Re}=\frac{2}{9}\left[2\Omega_\Lambda-(n-2)\Omega_m^0 f\left(\frac{\rho}{\rho_0}\right)\right]. \tag{57}$$

Or, equivalently,

$$\boxed{\Re''=(2/9)\Re\left[2\Omega_\Lambda-(n-2)\Omega_m^0 f\Re^{-n}\right]}. \tag{58}$$

The deceleration parameter becomes

$$q=-\frac{R\ddot R}{(\dot R)^2}=-\left(\frac{\Omega_\Lambda-0.5\Omega_m^0 f\Re^{-n}}{\Omega_\Lambda+\Omega_m^0 f\Re^{-n}}\right). \tag{59}$$

Equations (55) and (58) can be integrated with the use of the condition $\Re=1$, $\Re'=2/3$, and $\Omega_\Lambda=\Omega_\Lambda^0$ at $\tau=\tau_0$, provided f can be prescribed. Because the variability of G with cosmic time is not known, we will use a simple but plausible form:

$$f=(\tau/\tau_0)^m, \tag{60}$$

satisfying the condition $f=1$ at $\tau=\tau_0$. If $m=0$, we get the current constant G models. If $m>0$, G increases with cosmic time, and if $m<0$, it decreases. However, the solution for a decreasing G leads to negative values for Ω_Λ during the early phases of expansion implying negative vacuum/dark energy density, and therefore values of $m<0$ can be ruled out.

In the radiation-dominated universe, $n=4$. Using power-law solutions of the form $\Re=a\tau^p$ and $\Omega_\Lambda=b\tau^q$ and substituting in Eqs. (55) and (58), we get

$$qb\tau^{q-1}=-\Omega_m^0 a^{-4}\tau^{-4p}(m/\tau_0^m)\tau^{m-1} \tag{61}$$

and

$$p(p-1)\tau^{-2}=(4/9)\left[b\tau^q-\Omega_m^0 a^{-4}\tau^{-4p}(\tau/\tau_0)^m\right], \tag{62}$$

which immediately leads to

$$q=-2,$$

$$p=\frac{m+2}{4},$$

$$b=\frac{9mp}{16}=\frac{9m(m+2)}{64},$$

$$a^4=\frac{32\Omega_m^0}{9(m+2)\tau_0^m},$$

so that

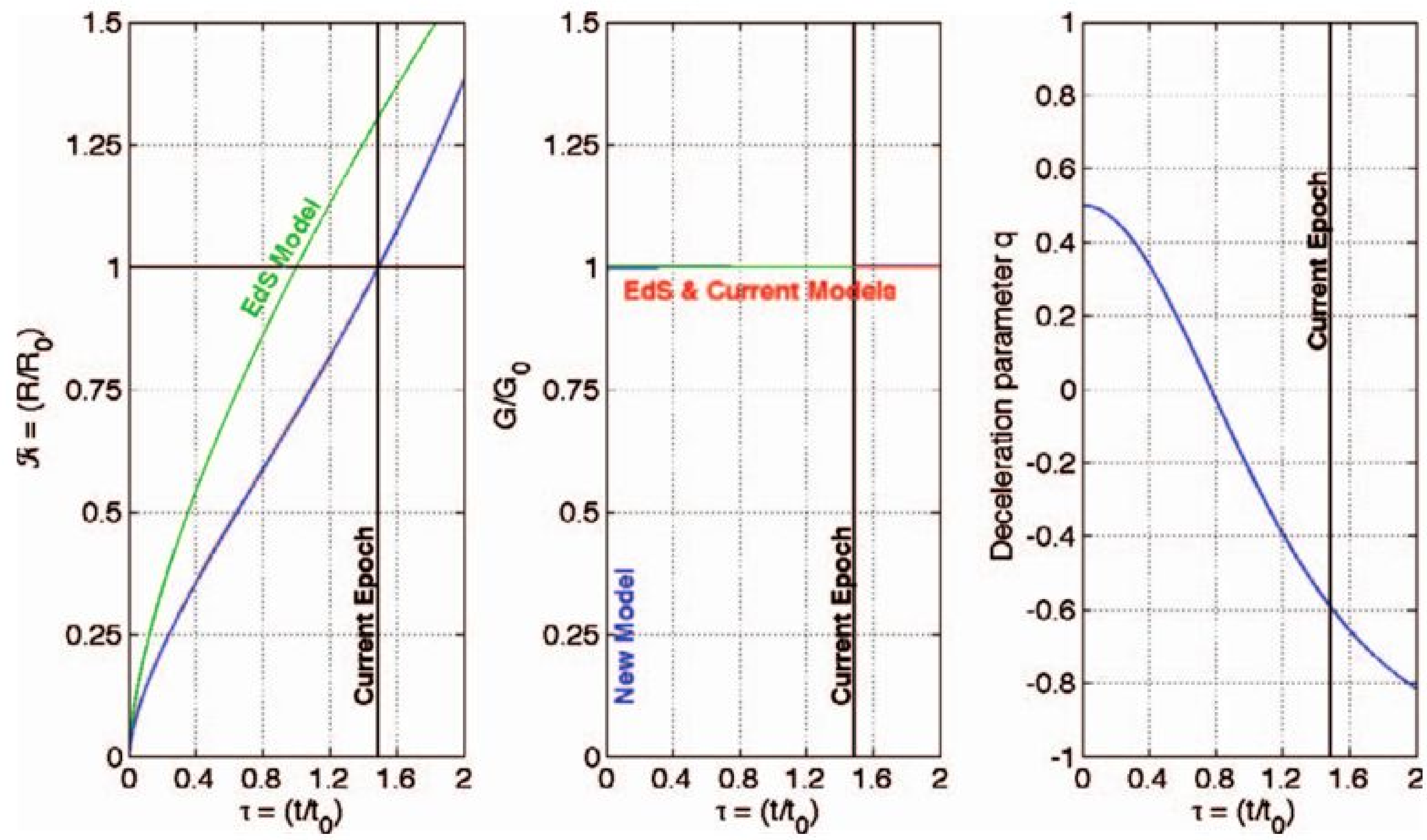

FIG. 1. (Color online) Variation of the normalized radius of the universe (left panel), the normalized gravitational constant (middle panel), and the deceleration parameter (right panel) with normalized cosmic time. The blue curves correspond to the increasing G model for $m=7.6\times10^{-4}$. The red curve corresponds to the current constant G models of the universe ($m=0$). The green curve corresponds to the Einstein–de Sitter model. The difference between the constant G and nonconstant G models is imperceptible in the left and right panels.

$$\Re=\left[32\Omega_m^0/9(m+2)\tau_0^m\right]^{1/4}\tau^{(m+2)/4}. \tag{63}$$

The question then is: What is the appropriate value of m? Because this question is hard to answer *a priori*, we will have to use observations to fix the value of m. One way to determine the correct value of m is to demand that the value of $\Re$ at which the matter-dominated solution [Eq. (58)] equals the radiation-dominated solution [Eq. (63)] be equal to the observed value[4] of 3×10^{-4}.

In the matter-dominated universe, $n=3$. Also, $\Omega_m^0=0.27$ and $\Omega_\Lambda^0=0.73$. With the use of these values, the third-order nonlinear set of ordinary differential equations, Eqs. (55) and (58), were solved for various values of m, with the ode23s Matlab routine for stiff equations. For each value of m, the value of $\Re$ at which this matched the radiation-dominated solution, Eq. (63), was determined. It turns out that for $m=7.6\times10^{-4}$, the two solutions match at $\Re\sim3\times10^{-4}$. Figure 1 shows the resulting variation of $\Re$, f, and q with cosmic time for this value of m (blue curve). The red line corresponds to the current constant $G(m=0)$ models. Clearly, the expansion of the universe is almost identical to that of current constant G models, because the value of m is so small. Figure 2 shows the corresponding variation of the cosmological parameter Ω_Λ and the normalized density of matter ρ/ρ_0 with time as the universe expands. It also shows the normalized acceleration of the universe as a function of cosmic time. The nonconstant G model solutions are indistinguishable from those for the current constant G models.

A salient aspect of the matter-dominated solution for the nonconstant G model is noteworthy. Although Ω_Λ remains positive definite and a decreasing function of τ, $\Re$ does not go to zero as $\tau\to0$. The nonzero value of $\Re$ at $\tau=0$ depends on the value of m. The smaller the value of m, the closer $\Re$ is to zero. For the chosen value of 7.6×10^{-4} for m, this value is roughly 3×10^{-4}. However, this does not matter, since as $\tau\to0$, the solution switches over to the radiation-dominated solution for which $\Re\to0$ as $\tau\to0$. Figure 3 (left panel) shows this.

The vacuum/dark energy density[7]

$$E_\Lambda=3\Lambda c^4/8\pi G=3\rho_{cr}c^2\Omega_\Lambda(G_0/G),$$

so that its normalized value is

$$\bar{E}_\Lambda=E_\Lambda/3\rho_{cr}c^2=\Omega_\Lambda/f. \tag{64}$$

The right panel of Fig. 3 shows the variation of $\bar{E}_\Lambda$ with cosmic time for $m=7.6\times10^{-4}$. It is noteworthy that $\bar{E}_\Lambda$ increases as $\tau\to0$.

For the radiation-dominated universe, from Eq. (43),

$$T=\left(\frac{\rho c^3}{4\sigma}\right)^{1/4}\Re^{-1} =\left(\frac{3H_0^2c^3}{32\pi G_0\sigma}\right)^{1/4}\left(\frac{9(m+2)\tau_0^m}{32}\right)^{1/4}\tau^{-[(m+2)/4]}. \tag{65}$$

Thus the solution is nearly identical to that of constant G models, Eq. (47), except for a very slight change in the power index and the proportionality constant. This means that the new solution allows for BBN but the exact time of BBN will be slightly different. The solution also requires

$$\Omega_\Lambda=[9m(m+2)/64]\tau^{-2}\sim0.002138\tau^{-2}, \tag{66}$$

so that the cosmological constant is a rapidly decreasing function of cosmic time. The normalized vacuum/dark energy density is

$$\bar{E}_\Lambda=\left[9m(m+2)\tau_0^m/64\right]\tau^{-(m+2)}\sim0.002139\tau^{-2.00076}. \tag{67}$$

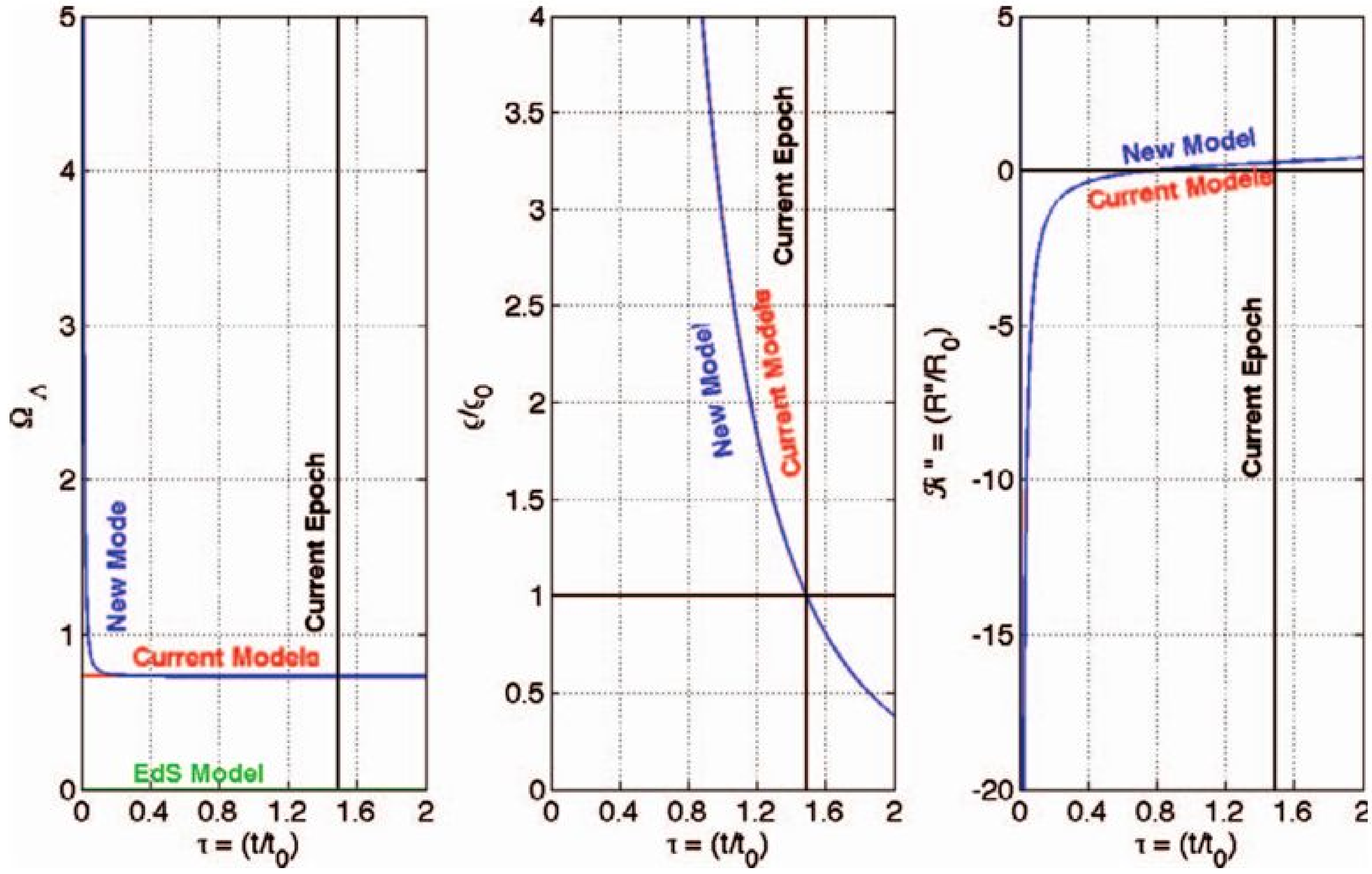

FIG. 2. (Color online) As in Fig. 1 but variation of Ω_Λ, ρ/ρ_0, and $\Re''$ with normalized cosmic time. The blue curve corresponds to nonconstant G model with m=7.6×10^{-4}. The red curve corresponds to the current constant G models of the universe (m = 0).

This behavior is physically more appealing than in current constant G models in which the vacuum/dark energy density remains unchanged as the universe expands. More importantly, $\bar{E}_\Lambda \to \infty$ as $\tau \to 0$, which is especially attractive, because it allows the vacuum/dark energy density E_Λ to approach Planck energy density $E_P = m_P \ell_P^{-3} c^2 \sim 4.5\times 10^{113}$ J m^{-3} (equivalently, $\bar{E}_P = 1.82\times 10^{122}$). For $m = 7.6\times 10^{-4}$, $\bar{E}_\Lambda \sim 0.00214\tau^{-2.0}$, and therefore immediately after the end of the inflation ($t \sim 10^{-32}$s, $\tau \sim 3.4\times 10^{-50}$), $\bar{E}_\Lambda \sim 1.85\times 10^{96}$, and

$$E_P/E_\Lambda \sim 10^{26}. \tag{68}$$

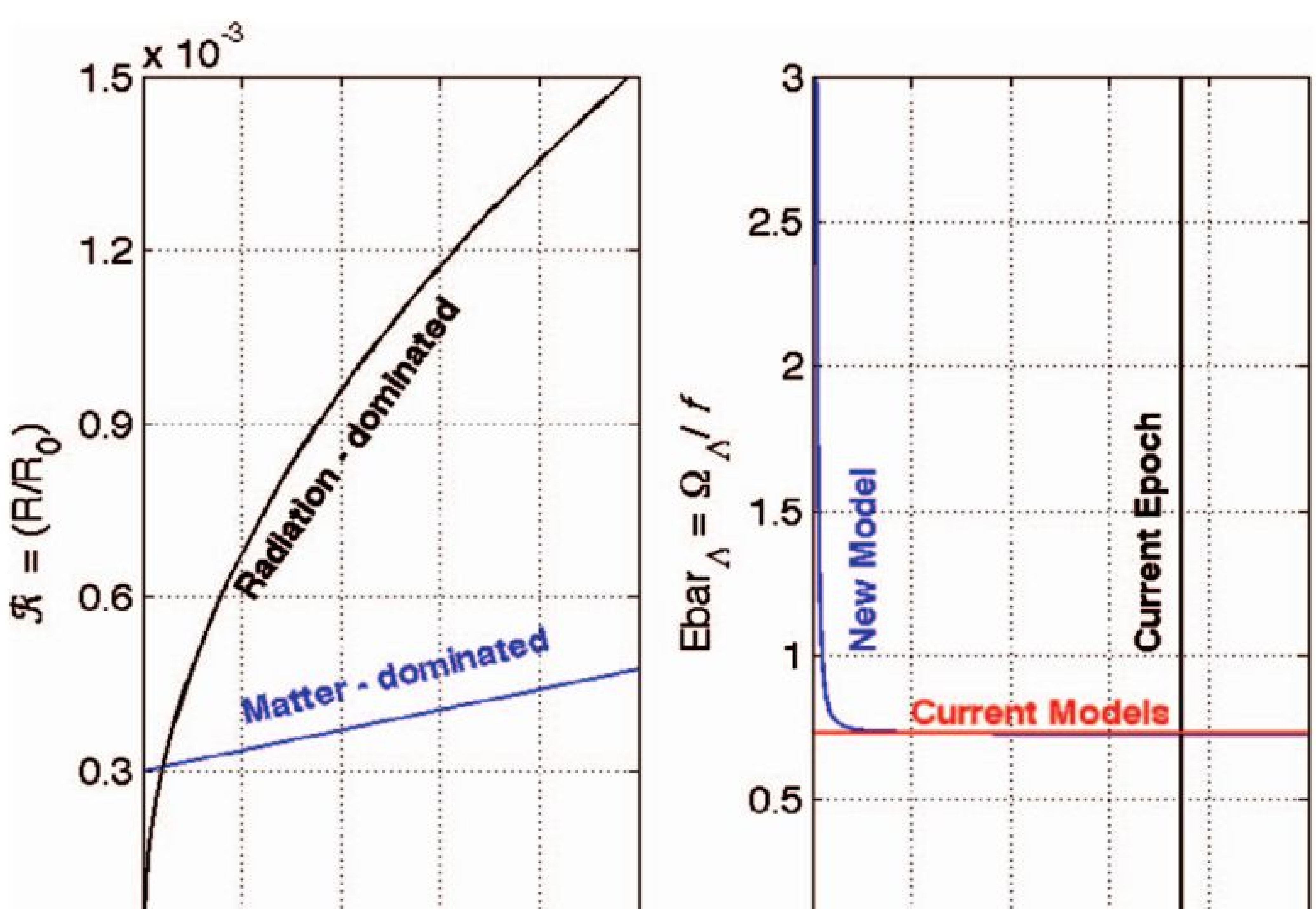

FIG. 3. (Color online) Left panel: normalized radius as a function of normalized cosmic time. The blue line is the matter-dominated solution [Eq. (58)] and the black curve is the radiation-dominated solution [Eq. (60)], both for ~1.055×10^{-34} m^{2}s^{-1} kg. Right panel: normalized vacuum energy density as a function of normalized cosmic time. The blue curve corresponds to the increasing G model with m=7.6×10^{-4}.

In contrast, in current constant G models, $E_\Lambda$stays unchanged at 1.853×10^{-9}J m^{-3} as the universe expands, and the ratio $E_P/E_\Lambda\sim2.5\times10^{122}$ remains incredibly large.[12] Note that

$$\dot{G}/G=m/t, \tag{69}$$

and hence G increases with time in the current model, rapidly in the early phases but very slowly at present, whereas some of the earlier models have G decreasing with time or, equivalently, $\dot{G}/G<0$.

V. CONCLUSIONS

Based on the theory of large numbers (see Funkhouser[12] for a modern discourse), Dirac[1] was one of the very first to suggest that G could be a function of cosmological time. In his 1974 paper,[2] he explored the implications of this to cosmological models. However, his cosmological model never got any traction, especially after Edward Teller stated that the Sun would have been too hot and the oceans would have boiled off a billion years ago.[13] However, Teller was wrong, a nonconstant G is plausible and over the intervening years, there has been sporadic interest in the topic. This article has explored a model that matches the radiation-dominated phase to the matter-dominated phase to infer a plausible variation of G with time. This nonconstant G model of the cosmos reproduces modern cosmological observations as well as the current constant G models. The postulated functional form for G is $G=G_0(\tau/\tau_0)^m$, where G_0 is the current value, τ is the cosmic time t normalized by the current age of the EdS universe t_o (9.207 Gyr), τ_0 ($\sim$1.489) is the current normalized age of the universe. The inferred value of m is approximately 0.00076. The small value of m implies that the variability of G is mostly confined to the very early phases of the big bang and G is very nearly constant at present. The most attractive aspect of the proposed model is that it calls for a nonconstant cosmological constant, thus avoiding having to postulate a nonchanging vacuum/dark energy density as the universe expands. Interestingly, the ratio of the Planck energy density to the vacuum/dark energy density is roughly 10^{26} immediately after the end of the inflation period, unlike current constant G models, where the ratio remains unbelievably large ($\sim10^{122}$). Recombination ($\Re^{-1}\sim1100$) occurs roughly 368 300 yr after the Big Bang, roughly the same time as in constant G models.

ACKNOWLEDGMENTS

Partial support from the FIRB Project RBFR08D825 (DECALOGO) is acknowledged.

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